Colored-noise-induced discontinuous transitions in ecosystems with Gompertz self-regulation

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A symbiotic ecosystem with Gompertz self-regulation is studied by means of the N-species Lotka-Volterra stochastic model. The influence of fluctuating environment on the carrying capacity of a population is modeled as the dichotomous noise. The study is a follow-up of previous investigations subjected to the generalized Verhulst self-regulation [1, 2].

In the framework of mean-field approximation the behavior of the solutions of the self-consistency equation for a stationary system is examined analytically in the full phase space of system parameters. Depending on the mutual interplay of symbiosis and competition of species, variation of noise parameters (amplitude, correlation time) can induce doubly unidirectional discontinuous transitions as well as single unidirectional discontinuous transitions of the mean population size. To our knowledge, the appearance of a noise-induced single unidirectional transitions (e.g., an increase in noise amplitude can cause a catastrophic fall in the size of populations, while by decreasing the noise amplitude no opposite transitions can occur) in models of ecosystems without extinction is a new noise-induced effect. We also show that, as compared with models with general Verhulst self-regulation mechanism (with an exponent $\beta > 1$), models with Gompertz law display a more sensitive response to environmental fluctuations.

This novel feature of symbiotic ecosystems can provide a possible scenario for some catastrophic shifts of population sizes observed in nature [3], e.g., in the case of coral reefs, where symbiosis is essential. Finally, we believe that the model and the results discussed here are of interest also in other fields where Gompertz self-regulation is relevant in system modeling, e.g., in market forecasting, in oncology, in bioeconomics, and in the theory of cultural diffusion.

1. R. Mankin, A. Ainsaar, A. Haljas, and E. Reiter, Phys. Rev. E **65**, 051108 (2002);
2. R. Mankin, A. Sauga, A. Ainsaar, A. Haljas, and K. Paunel, Phys. Rev. E **69**, 061106 (2004).
3. M. Scheffer, S. Carpenter, J. A. Foley, C. Folke, and B. Walker, Nature (London) **413**, 591 (2001).